## Large deviations and ruin probability for random walks with heavy-tailed step sizes

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Over the last few years, the stochastic recurrence equation (SRE) $X_t = A_t X_{t-1} + B_t$ has attracted quite a few attention, where $((A_t, B_t))$ is an iid sequence of random variables $A_t$ and $B_t$. This is, among others, due to the fact that the popular squared GARCH processes can be embedded in such an equation. It was observed by Kesten (1973) and Goldie (1991) that light-tailed input $(A_t, B_t)$ can cause the output $(X_t)$, the stationary solution to SRE, to have marginal distributions with regularly varying tails,

In contrast to the latter approach, we consider heavy-tailed (regularly varying with index $\kappa > 1$) $B_t$'s. They cause $(X_t)$ to have regularly varying finite-dimensional distributions. Under some mild conditions, $(X_t)$ is strongly mixing with geometric rate. We study the large deviations of the random walk $S_n = X_1 + \cdots + X_n$ from its mean. Typically,

$$\sup_{x \geq x_n} \left| \frac{P(S_n - ES_n > x)}{n\, P(X_1 > x)} - c \right| \to 0$$

for some positive $c > 0$ and suitable $x_n \to \infty$. This shows that the tail $P(S_n - ES_n > x)$ for sufficiently high thresholds $x$ is determined by one large value $X_i$. In the case of iid $X_i$'s one has $c = 1$. In the case of a SRE, $c$ depends on the distribution of $A_1$ and is different from 1. Large deviations are closely related to ruin probabilities, i.e., $\psi(u) = P(\sup_{n\geq 1}(S_n - ES_n - \mu\, n) > u)$ for positive $\mu$ and $u \to \infty$. We show the result that $\psi(u) \sim C\, (\kappa - 1)^{-1} \mu^{-1} u\, P(X_1 > u)$ for some constant $c \neq C \neq 1$. This is analogous to the case of iid $X_t$, where $C = 1$.

These result indicate how the dependence in the sequence $(X_t)$ determines the clustering behavior of high-level exceedances of the random walk $(S_n - ES_n)$. It is one step in the direction of understanding this behavior for random walks with step sizes generated by a general stationary Markov chain.