The Bose gas beyond mean field

We consider Bose particles interacting by means of a Kac potential with range controlled by a scaling parameter γ. As γ tends to zero the range is driven to infinity, thus providing the mean field limit of the Bose gas. The mean field theory is fully understood, but the leading corrections for $\gamma \neq 0$ are not known. We apply to this problem the recently developped technique of quantum Mayer graphs. Quantum Mayer graphs arise when the Gibbs statistical weight is represented by the Feynman-Kac path integral with proper treatment of quantum statistics. It is seen that the summation of tree-graphs reproduces the mean field theory, but the powerful tools of the Mayer diagrammatics offers (as in the classical case) the possibility of deeper insight and calculations for the interacting system. We discuss problems that arise when one wants to go beyond mean field and establish various links with the theory of polymers.