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Scattering by Magnetic Fields

Abstract

Part 1 On the mathematical theory of the Aharonov-Bohm effect.

We consider the Schrödinger operator $H = (i\nabla + A)^2$ in the space $L_2(\mathbb{R}^2)$ with a magnetic potential $A(x) = a(\hat{x})(-x_2, x_1)|x|^{-2}$, where a is an arbitrary function on the unit circle. Our goal is to study spectral properties of the corresponding scattering matrix $S(\lambda)$, $\lambda > 0$. We obtain its stationary representation and show that its singular part (up to compact terms) is a pseudodifferential operator of zero order whose symbol is an explicit function of a. We deduce from this result that the essential spectrum of $S(\lambda)$ does not depend on λ and consists of two complex conjugated and perhaps overlapping closed intervals of the unit circle. Finally, we calculate the diagonal singularity of the scattering amplitude (kernel of $S(\lambda)$ considered as an integral operator). In particular, we show that for all these properties only the behaviour of a potential at infinity is essential. The preceeding papers on this subject treated the case $a(\hat{x}) = \text{const}$ and used the separation of variables in the Schrödinger equation in the polar coordinates. This technique does not of course work for arbitrary a. From analytical point of view, our paper relies on some modern tools of scattering theory and well-known properties of pseudodifferential operators.

Part 2 There is no Aharonov-Bohm effect in dimension three.

Consider the scattering amplitude $s(\omega, \omega'; \lambda)$, $\omega, \omega' \in \mathbb{S}^2$, $\lambda > 0$, corresponding to an arbitrary three-dimensional short-range magnetic field $B(x)$. The magnetic potential $A^{(tr)}(x)$ such that $\operatorname{curl} A^{(tr)}(x) = B(x)$ and such that $< A^{(tr)}(x), x >= 0$ decays at infinity as $|x|^{-1}$ only. Nevertheless, we show that the structure of $s(\omega, \omega'; \lambda)$ is the same as for short-range magnetic potentials. In particular, the leading diagonal singularity $s_0(\omega, \omega')$ of $s(\omega, \omega'; \lambda)$ is the Dirac function. Thus, up to the diagonal Dirac function, the scattering amplitude has only a weak singularity in the forward direction and hence scattering is essentially of short-range nature. This is qualitatively different from the two-dimensional case where $s_0(\omega, \omega')$ is a linear combination of the Dirac function and of a singular denominator, that is the Aharonov-Bohm effect occurs. Our approach relies on a construction of a special gauge, adapted to a given magnetic field $B(x)$, such that the corresponding magnetic potential $A(x)$ is also short-range.