ABSTRACTS FROM THE MINI-WORKSHOP ON STOCHASTICS

ABSTRACT. This short note contains the abstracts from the mini-workshop held August 4-6, 1998 at the Department of Mathematical Sciences, University of Aarhus.

For further information, contact the speakers (some email addresses are given) or MaPhySto.

LIST OF SPEAKERS

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Superposition of Ornstein-Uhlenbeck type processes.

Superposition of independent Gaussian Ornstein-Uhlenbeck (OU) processes has been considered in connection with a model of neural response (Walsh, 1981) and also from a pure mathematics point of view (Csáki, Csörgõ, Lin and Révész, 1991; Lin, 1995). Superposition of other, that is non-Gaussian, types of OU processes were introduced in turbulence (Barndorff-Nielsen, Jensenand Sørensen, 1990, 1993, 1998) and, more recently, in mathematical finance (Barndorff-Nielsen, 1998; Barndorff-Nielsen and Shephard, 1998a,b) in order to model distributional behaviour and timewise dependence structures typically found in observational series of, respectively, velocity differences and log asset returns. Existence and properties of OU processes with given one-dimensional marginal laws are discussed in some detail and for the case where the law is either inverse Gaussian or normal inverse Gaussian the precise nature of the innovation process, also termed the *background driving Lévy process* (BDLP), is derived. The latter two types of OU processes are shown to yield flexible and analytically tractable models. A rigorous approach to the definition of superposition of a continuum of independent OU processes is outlined.

References

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The Malliavin derivative for random distributions.

We extend the Malliavin derivative to random distributions which does not admit a chaos expansion with square integrable kernels. The space of Hida distributions is considered, were the random distributions have a formal expansion with kernels belonging to the tempered distributions. We define the Malliavin derivative for Hida distributions and discuss some properties of it.

As an example we calculate the Malliavin derivative of white noise, the time derivative of Brownian motion.

Jørgen Hoffmann-Jørgensen (Aarhus)

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Optimal stopping times.

Let $X_1, \ldots, X_d$ be the return sequence in a finite or infinite number d of games which are adapted the filter $\mathcal{F}_1 \subseteq \ldots \subseteq \mathcal{F}_d$. In the theory of optimal stopping we are confronted with the problem of integrability of X_τ when τ is a stopping time. In the literature this problem is usually dealt with by considering stopping times τ satisfying $E|X_\tau| < \infty$ or $EX_\tau^- < \infty$ or τ is bounded. In the talk, I shall show how the use of lower and upper (conditional) expectations can be used to overcome the problem without making any integrability restrictions on the stopping times τ or on the returns (X_n) (the returns may even take the values

$\pm\infty$ with positive probability, which is convenient in some examples). The main stream in the literature has been the study of optimality and computability of the global Snell stopping time. In the talk I shall turn the view upside down and consider a given stopping time and study its optimality properties. In particular, I shall consider

- *passage times;* i.e. the first time that (X_n) exceeds a given sequence (Γ_n) of random variables (for instance, solutions to the backwards equations: $\Gamma_n = X_n \vee E_*(\Gamma_{n+1}|\mathcal{F}_n)$ a.s.)
- *risk averse Snell stopping times;* i.e. the optimal stopping problem for a restricted class of "permissible" stopping times (for instance, the set of stopping times τ such that your loss X_ρ^- is not too excessive at any time $\rho \leq \tau$)
- *admissible kernels;* which is a powerful method of improving the performance of a given stopping time
- *unimodal stopping times;* i.e. stopping times τ such that (X_n) "increases" up to time τ and "decreases" after time τ
- *randomization of stopping times;* which is a powerful method of removing redundant information from the filter $(\mathcal{F}_n)$
- *simple stopping times;* i.e. the first time that (X_n) exceeds a given sequence (c_n) of (extended) real numbers.

MARTIN JACOBSEN (COPENHAGEN)

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Discretely observed diffusions: Classes of estimating functions and criteria for choosing good ones.

The problem considered is that of estimating the parameters of a discretely observed, ergodic homogeneous diffusion. Since the maximum likelihood estimator (MLE) is only available in exceptional cases, various classes of estimating functions (ef's) have been studied, including martingale ef's (Bibby and Sørensen) and simple, explicit ef's (Kessler). In the talk some new classes are proposed, including the explicit, transition dependent ef's. These classes are derived using ideas involving time reversal. Further, the asymptotic theory of the estimators obtained from general ef's is surveyed and then used to discuss optimality criteria involving the properties of the estimators when the observations are either close in time or very far apart, using suitable interpolations for the in-between cases. This approach, which leads to easily verifiable conditions, is suggested since the more standard method of e.g. minimizing the asymptotic variance within a class of estimators will prove impractical for theoretical and/or numerical reasons.

Thomas Mikosch (Groningen)

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The extremes of the periodogram.

This talk is about joint work with R.A. Davis, P. Kokoszka, G. Samorodnitsky and S.I. Resnick.

In the literature one can often find the vague statement that the periodogram ordinates $I_{n,Z}(\lambda_j)$ of an iid sequence (Z_t) when evaluated at the Fourier frequencies $\lambda_j = 2\pi j/n$, $j = 1, \ldots, [n/2]$ "behave like an iid exponential sequence". If the underlying iid sequence is Gaussian the $I_{n,Z}(\lambda_j)$'s are iid exponential. But does this statement remain valid, if we have non-Gaussian data?

In the course of the talk we consider various functions of the $I_{n,Z}(\lambda_j)$'s. This includes their maximum, the upper order statistics and the point process constructed from those ordinates. Using a large deviation result by U. Einmahl (1989), one can show that these functions have the same limit behaviour as if one replaced the $I_{n,Z}(\lambda_j)$'s with an iid exponential sample. If $EZ_t^2 < \infty$, the empirical distribution function of the $I_{n,Z}(\lambda_j)$'s satisfies a Glivenko–Cantelli result with exponential limit (this was proved by Freedman and Lane (1981), we have a different proof based on Hermann Weyl's (1916) theorem about the uniform distribution of a sequence of real numbers), but the central limit theorem does not seem to work for the corresponding empirical process since higher–order moment characteristics seem to play a rôle. We also mention that weighted sums of the $I_{n,Z}(\lambda_j)$'s (such as the empirical spectral distribution function, MLE– and Whittle–type estimates of the parameters of ARMA processes) depend on the higher–order moment structure of the Z_t's, so that the analogy with an iid exponential sequence may fail.

One has a totally different asymptotic behaviour for functions of the $I_{n,Z}(\lambda_j)$'s when the Z_t's have infinite variance. An indication of this fact is that a finite number of the $I_{n,Z}(\lambda_j)$'s converges to a function of an infinite variance stable process, the empirical distribution based on the $I_{n,Z}(\lambda_j)$'s converges to a random measure (this is a result by Freedman and Lane (1980)) and the extremes have some non–standard limit behaviour as well.

GORAN PESKIR (AARHUS)

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On Brownian motion in a force field.

With a view to applications in financial mathematics, we consider the problem of Brownian motion under influence of an external force field. In this context we discuss contributions by Einstein, Smoluchowski, Langevin, Ornstein, Uhlenbeck, Doob, Nelson, etc.

ALBERT N. SHIRYAEV (MOSCOW)

On the laws of the "downfalls" in Brownian motion.

This talk is about joint work with R. Douady and M. Yor.

For a standard Brownian motion $B = (B_t)_{t\leq 1}$ we consider the following functionals:

$$D = \max_{0\leq t\leq t'\leq 1} (B_t - B_{t'})$$

(i.e. maximal value of the downfalls),

$$D_1 = B_\sigma - \min_{\sigma\leq t'\leq 1} B_{t'}$$

(i.e. value of the downfall from absolute maximum $B_\sigma = \max_{0\leq t\leq 1} B_t$ to the partial minimum $\min_{\sigma\leq t'\leq 1} B_{t'}$),

$$D_2 = \sup_{0\leq t\leq \sigma'} B_t - B_{\sigma'}$$

(i.e. value of the downfall from the partial maximum $\max_{0\le t\le\sigma'} B_t$ to the absolute minimum $B_{\sigma'} = \min_{0\le t'\le 1} B_{t'}$).

Our aim is to describe probability laws of the D, D_1, D_2. We prove, for example, that

$$D \stackrel{law}{=} \sup_{0\le t\le 1} |B_t|$$

and the probability density $f_{D_1}(x)$ of D_1 is given by the formula

$$f_{D_1}(x) = \sqrt{\frac{8}{\pi}} \sum_{k=1}^{\infty} (-1)^{k-1} e^{-\frac{k^2x^2}{2}}.$$

We also show that

$$ED = \sqrt{\frac{\pi}{2}} \qquad (= 1.2533..)$$

and

$$ED_1 = \sqrt{\frac{8}{\pi}} \ln 2 \qquad (= 1.1061..).$$