Sequential Testing Problems for Poisson Processes

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Abstract

We present the explicit solution of the Bayesian and variational problem of sequential testing of two simple hypotheses about the intensity of an observed Poisson process. The method of proof consists of reducing the initial problem to a free-boundary differential-difference Stephan problem, and solving the latter by use of the principles of smooth and continuous fit. A rigorous proof of the optimality of the Wald's sequential probability ratio test in the variational formulation of the problem is obtained as a consequence of the solution of the Bayesian problem.

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Key words and phrases: Sequential testing, Bayes decision rule, Poisson process, SPRT (sequential probability ratio test), optimal stopping, a free-boundary differential-difference Stephan problem, the principles of continuous and smooth fit, point (counting) (Cox) process, measure of jumps and its compensator, Itô's formula.

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